OSCILACIJE

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1. Dva matematička klatna jednovremeno osciluju. Za isto vreme jedno napravi 15 oscilacija a drugo 10 oscilacija. Nađi odnos dužina klatna.

$n_1 = 15$

$n_2 = 10$

$\frac{l_1}{l_2} = ?$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\frac{l}{g} = \frac{T^2}{4\pi^2}$$

$$l = \frac{gT^2}{4\pi^2}$$

$$l = \frac{g\left(\frac{t}{n}\right)^2}{4\pi^2}$$

$$l = \frac{g\frac{t^2}{n^2}}{4\pi^2}$$

$$l = \frac{gt^2}{4\pi^2 n^2}$$

$$l_1 = \frac{gt^2}{4\pi^2 {n_1}^2} \qquad l_2 = \frac{gt^2}{4\pi^2 {n_2}^2}$$

$$\frac{l_1}{l_2} = \frac{\frac{gt^2}{4\pi^2 {n_1}^2}}{\frac{gt^2}{4\pi^2 {n_2}^2}}$$

$$\frac{l_1}{l_2} = \frac{gt^2 4\pi^2 {n_2}^2}{gt^2 4\pi^2 {n_1}^2}$$

$$\frac{l_1}{l_2} = \frac{{n_2}^2}{{n_1}^2} = \frac{10^2}{15^2} = \frac{100}{225} = \frac{4}{9}$$

$$l_1 = \frac{4}{9} l_2$$

2. U nepokretnoj kabini lifta osciluje matematičko klatno sa periodom T_0. Kolikim ubrzanjem treba da se podiže lift da bi period bio $\frac{T_0}{2}$?

$$T_0 = 2\pi\sqrt{\frac{l}{g}} \qquad T = 2\pi\sqrt{\frac{l}{g+a}}$$

$$T = \frac{T_0}{2}$$

a =?

$$T = \frac{T_0}{2}$$

$$2\pi\sqrt{\frac{l}{g+a}} = \frac{2\pi\sqrt{\frac{l}{g}}}{2}$$

$$2\pi\frac{\sqrt{l}}{\sqrt{g+a}} = \pi\sqrt{\frac{l}{g}}$$

$$2\pi\frac{\sqrt{l}}{\sqrt{g+a}} = \pi\frac{\sqrt{l}}{\sqrt{g}}$$

$$2\frac{1}{\sqrt{g+a}} = \frac{1}{\sqrt{g}}$$

$$\frac{2}{\sqrt{g+a}} = \frac{1}{\sqrt{g}}$$

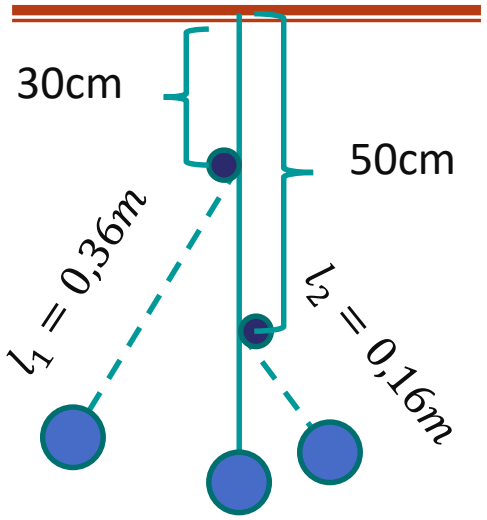
$$2\sqrt{g} = \sqrt{g+a}$$

$$4g = g + a$$

$$a = 4g - g$$

$$a = 3g$$

3. Matematičko klatno dužine 66 cm okačeno je tako da njegova nit prolazi između dva eksera od kojih se jedan nalazi na 30 cm, a drugi na 50 cm ispod tačke vešanja. Naći period oscilacija ovog klatna.



$$l_1 = 66cm - 30cm = 36cm = 0{,}36m$$

$$l_1 = 66cm - 50cm = 16cm = 0{,}16m$$

$$T = \frac{T_1}{2} + \frac{T_2}{2}$$

$$T = \frac{2\pi\sqrt{\frac{l_1}{g}}}{2} + \frac{2\pi\sqrt{\frac{l_2}{g}}}{2}$$

$$T = \pi\sqrt{\frac{l_1}{g}} + \pi\sqrt{\frac{l_2}{g}}$$

$$T = 3{,}14\sqrt{\frac{0{,}36m}{10\frac{m}{s^2}}} + 3{,}14\sqrt{\frac{0{,}16m}{10\frac{m}{s^2}}}$$

$$T = 3{,}14\sqrt{0{,}036s^2} + 3{,}14\sqrt{0{,}016s^2}$$

$$T \approx 3{,}14 \cdot 0{,}19s + 3{,}14 \cdot 0{,}13s$$

$$T \approx 0{,}6s + 0{,}4s$$

$$T \approx 1s$$

4. Od konca dužine 3,15 metara treba da se naprave 3 matematička klatna, pri čemu je period jednog klatna dva puta manji od drugog i dva puta veći od trećeg. Kolike su dužine tih klatna?

$l = 3{,}15m$

$T_1 = \frac{T_2}{2} \Rightarrow T_2 = 2T_1$

$T_1 = 2T_3 \Rightarrow T_3 = \frac{T_1}{2}$

$l_1 = ?\quad l_2 = ?\quad l_3 = ?$

$T = 2\pi\sqrt{\frac{l}{g}}$

$\frac{l}{g} = \frac{T^2}{4\pi^2}$

$l = \frac{gT^2}{4\pi^2}$

$l_1 = \frac{gT_1{}^2}{4\pi^2}$

$l_2 = \frac{gT_2{}^2}{4\pi^2}$

$l_3 = \frac{gT_3{}^2}{4\pi^2}$

$l = l_1 + l_2 + l_3$

$l = \frac{gT_1{}^2}{4\pi^2} + \frac{gT_2{}^2}{4\pi^2} + \frac{gT_3{}^2}{4\pi^2}$

$l = \frac{g}{4\pi^2}\left(T_1{}^2 + T_2{}^2 + T_3{}^2\right)$

$l = \frac{g}{4\pi^2}\left(T_1{}^2 + (2T_1)^2 + (\frac{T_1}{2})^2\right)$

$l = \frac{g}{4\pi^2}\left(T_1{}^2 + 4T_1{}^2 + \frac{T_1{}^2}{4}\right)$

$l = \frac{g}{4\pi^2}T_1{}^2\left(1 + 4 + \frac{1}{4}\right)$

$l = \frac{g}{4\pi^2}T_1{}^2 5{,}25$

$\frac{4\pi^2 l}{5{,}25\ g} = T_1{}^2$

$T_1{}^2 = \frac{4\pi^2 l}{5{,}25\ g}$

$T_1{}^2 = \frac{4 \cdot 3{,}14^2 \cdot 3{,}15m}{5{,}25 \cdot 10\frac{m}{s^2}}$

$T_1{}^2 = \frac{124m}{52{,}5\frac{m}{s^2}} = 2{,}36s^2$

$T_1 = \sqrt{2{,}36s^2} = 1{,}54s$

$T_2 = 2T_1 = 2 \cdot 1{,}54s$

$T_2 = 3{,}08s$

$T_3 = \frac{T_1}{2} = \frac{1{,}54s}{2}$

$T_3 = 0{,}77s$

$l_1 = \frac{gT_1{}^2}{4\pi^2} = \frac{10\frac{m}{s^2}(1{,}54s)^2}{4 \cdot 3{,}14^2}$

$l_1 = \frac{23{,}7m}{39{,}4} = 0{,}6m$

$l_2 = \frac{gT_2{}^2}{4\pi^2} = \frac{10\frac{m}{s^2}(3{,}08s)^2}{4 \cdot 3{,}14^2}$

$l_2 = \frac{94{,}9m}{39{,}4} = 2{,}4m$

$l_3 = \frac{gT_3{}^2}{4\pi^2} = \frac{10\frac{m}{s^2}(0{,}77s)^2}{4 \cdot 3{,}14^2}$

$l_2 = \frac{5{,}929m}{39{,}4} = 0{,}15m$

5. Koliku masu treba dodati tegu mase m koje osciluje na opruzi da se period poveća tri puta?

$$T = 2\pi\sqrt{\frac{m}{g}}$$

$$T_1 = 2\pi\sqrt{\frac{m+m_1}{g}}$$

$$T_1 = 3T$$

$$2\pi\sqrt{\frac{m+m_1}{g}} = 3\cdot 2\pi\sqrt{\frac{m}{g}}$$

$$\sqrt{\frac{m+m_1}{g}} = 3\sqrt{\frac{m}{g}}$$

$$\frac{\sqrt{m+m_1}}{\sqrt{g}} = 3\frac{\sqrt{m}}{\sqrt{g}}$$

$$\sqrt{m+m_1} = 3\sqrt{m}$$

$$m+m_1 = 9m$$

$$m_1 = 9m - m$$

$$m_1 = 8m$$

6. Elastična opruga se istegne za 1,5 cm kad o nju obesimo telo mase 1 kg. Kolika je frekvencija oscilovanja sistema ako o oprugu obesimo telo mase 10 kg?

$\Delta l = 1{,}5cm = 0{,}015m$

$m_1 = 1kg$

$k = \frac{F_t}{\Delta l} = \frac{m_1 \cdot g}{\Delta l} = \frac{1kg \cdot 10\frac{N}{kg}}{0{,}015m} \approx 666{,}67\frac{N}{m}$

$m = 10kg$

$\nu = ?$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2 \cdot 3{,}14\sqrt{\frac{10kg}{666{,}67\frac{N}{m}}}$

$T = 6{,}28 \cdot 0{,}1225\ s$

$T \approx 0{,}77s$

$\nu = \frac{1}{T}$

$\nu = \frac{1}{0{,}77s}$

$\nu \approx 1{,}3Hz$

7. Telo mase 500 g pri oscilovanju duž horizontalnog pravca u jednom trenutku ima brzinu 20 m/s dok joj je potencijalna energija 300 J. Kolikom brzinom prolazi telo kroz ravnotežni položaj?

$m = 500g = 0{,}5kg$

$v = 20\frac{m}{s}$

$E_p = 300J$

$$E_k = \frac{m \cdot v^2}{2} = \frac{0{,}5kg \cdot (20\frac{m}{s})^2}{2} = \frac{0{,}5kg \cdot 400(\frac{m}{s})^2}{2} = 100J$$

$$E_u = E_k + E_p$$

$$E_u = 100J + 300J = 400J = E_{k\,rp}$$

$$E_{k\,rp} = \frac{m \cdot {v_{rp}}^2}{2}$$

$${v_{rp}}^2 = \frac{2 \cdot E_{k\,rp}}{m}$$

$${v_{rp}}^2 = \frac{2 \cdot 400J}{0{,}5kg}$$

$${v_{rp}}^2 = 1600\frac{J}{kg}$$

$$v_{rp} = \sqrt{1600\frac{J}{kg}}$$

$$v_{rp} = 40\frac{m}{s}$$