

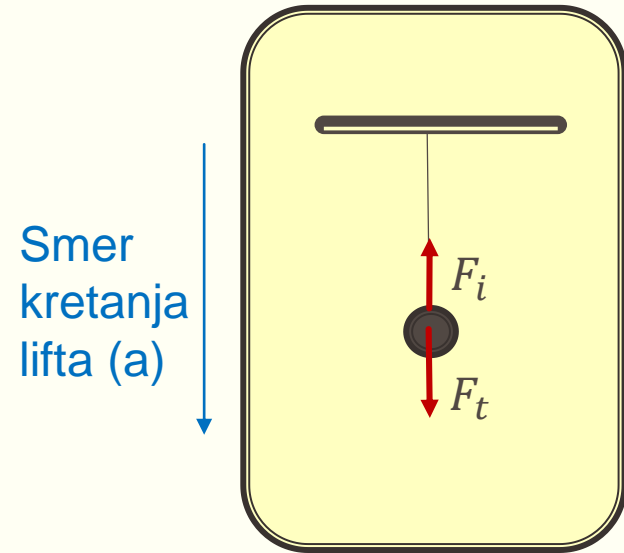


MATEMATIČKO KLATNO

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Matematičko klatno u liftu



$$F_r = F_t - F_i$$

$$m \cdot g_1 = m \cdot g - m \cdot a$$

$$m \cdot g_1 = m \cdot (g - a)$$

$$g_1 = g - a$$

$$F_t = m \cdot g$$

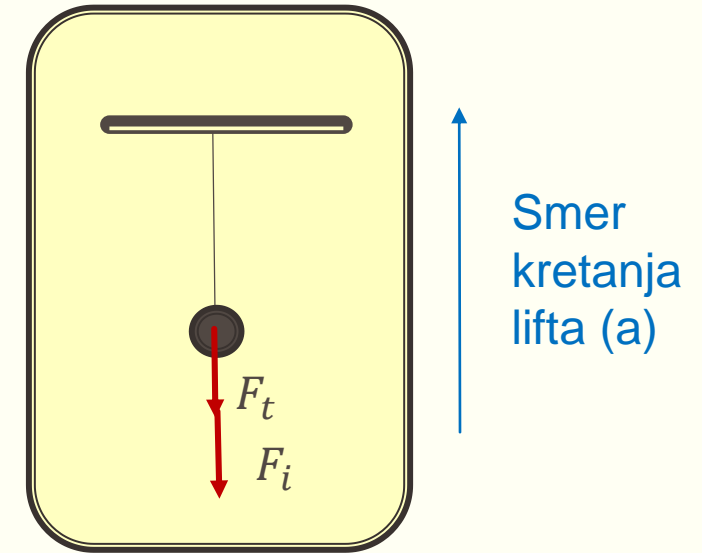
$$F_i = m \cdot a$$

$$F_r = m \cdot g_1$$

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \rightarrow \quad T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g - a}}$$



$$F_r = F_t + F_i$$

$$m \cdot g_1 = m \cdot g + m \cdot a$$

$$m \cdot g_1 = m \cdot (g + a)$$

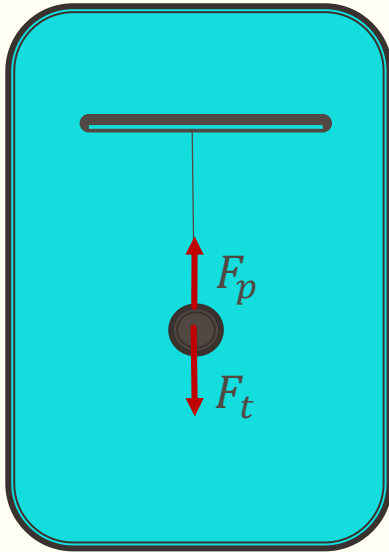
$$g_1 = g + a$$

$$T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g + a}}$$

Matematičko klatno u tečnosti

$\rho_0 < \rho$



$$F_r = F_t - F_p$$

$$m \cdot g_1 = m \cdot g - m \cdot g \cdot \frac{\rho_0}{\rho}$$

$$m \cdot g_1 = m \cdot g \cdot \left(1 - \frac{\rho_0}{\rho}\right)$$

$$g_1 = g \cdot \left(1 - \frac{\rho_0}{\rho}\right)$$

ρ_0 - gustina tečnosti ρ - gustina tela

$$F_t = m \cdot g$$

$$F_p = \rho_0 \cdot g \cdot V = \rho_0 \cdot g \cdot \frac{m}{\rho}$$

$$F_p = m \cdot g \cdot \frac{\rho_0}{\rho}$$

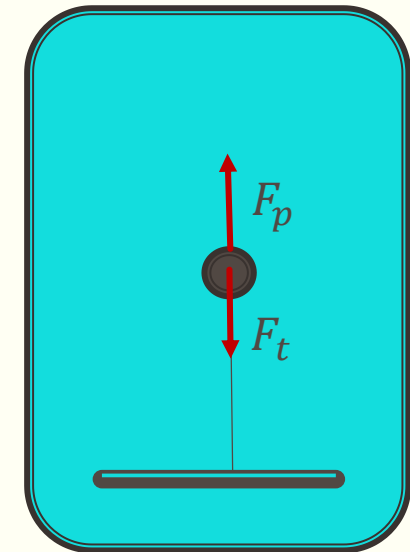
$$F_r = m \cdot g_1$$

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \Rightarrow \quad T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g \cdot \left(1 - \frac{\rho_0}{\rho}\right)}}$$

$\rho_0 > \rho$



$$F_r = F_p - F_t$$

$$m \cdot g_1 = m \cdot g \cdot \frac{\rho_0}{\rho} - m \cdot g$$

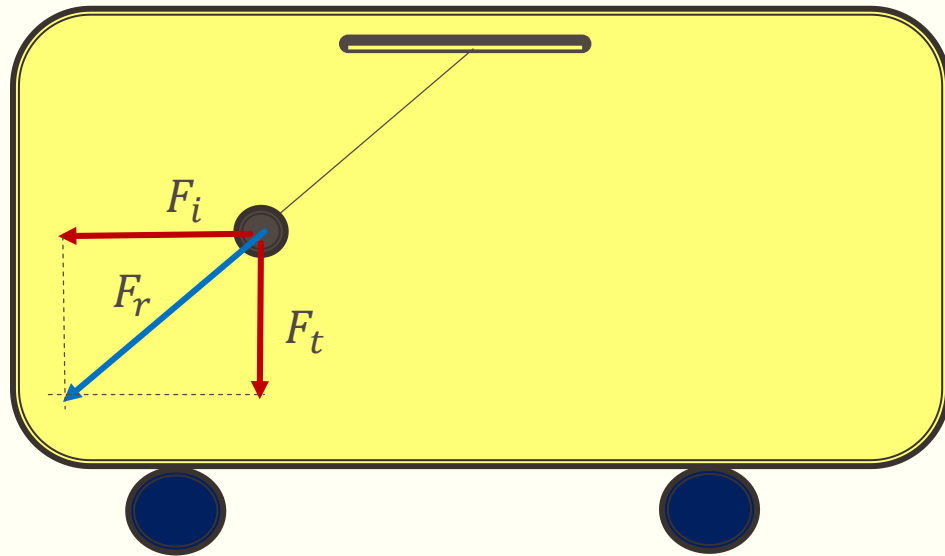
$$m \cdot g_1 = m \cdot g \cdot \left(\frac{\rho_0}{\rho} - 1\right)$$

$$g_1 = g \cdot \left(\frac{\rho_0}{\rho} - 1\right)$$

$$T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{g \cdot \left(\frac{\rho_0}{\rho} - 1\right)}}$$

Matematičko klatno u vagonu, automobilu, kamionu



$$F_t = m \cdot g$$

$$F_i = m \cdot a$$

$$F_r = m \cdot g_1$$

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \rightarrow \quad T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$F_r^2 = F_t^2 + F_i^2$$

$$(m \cdot g_1)^2 = (m \cdot g)^2 + (m \cdot a)^2$$

$$m^2 \cdot g_1^2 = m^2 \cdot g^2 + m^2 \cdot a^2$$

$$m^2 \cdot g_1^2 = m^2 \cdot (g^2 + a^2)$$

$$g_1^2 = g^2 + a^2$$

$$g_1 = \sqrt{g^2 + a^2}$$

$$T = 2\pi \sqrt{\frac{l}{g_1}}$$

$$T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

Matematičko klatno dužine 0,11m osciluje u liftu. Odredi period i frekvenciju klatna ako se lifr kreće naviše ubrzanjem $1 \frac{m}{s^2}$.

$$l = 0,11m$$

$$a = 1 \frac{m}{s^2}$$

$$T = ?$$

$$\nu = ?$$

$$T = 2\pi \sqrt{\frac{l}{g+a}}$$

$$\nu = \frac{1}{T}$$

$$T = 2 \cdot 3,14 \sqrt{\frac{0,11m}{10 \frac{m}{s^2} + 1 \frac{m}{s^2}}}$$

$$\nu = \frac{1}{0,628s}$$

$$\nu \approx 1,6Hz$$

$$T = 6,28 \sqrt{\frac{0,11m}{11 \frac{m}{s^2}}}$$

$$T = 6,28 \sqrt{0,01s^2}$$

$$T = 6,28 \cdot 0,1s$$

$$T = 0,628s$$

Matematičko klatno dužine 1m osciluje u vodi. Odredi period i frekvenciju klatna ako je gustina tela $10000 \frac{kg}{m^3}$.

$$l = 1m$$

$$\rho = 10000 \frac{kg}{m^3}$$

$$T = ?$$

$$\nu = ?$$

$$\rho_0 = 1000 \frac{kg}{m^3}$$

$$T = 2\pi \sqrt{\frac{l}{g \cdot (1 - \frac{\rho_0}{\rho})}}$$

$$T = 2 \cdot 3,14 \sqrt{\frac{1m}{10 \frac{m}{s^2} \cdot (1 - \frac{1000 \frac{kg}{m^3}}{10000 \frac{kg}{m^3}})}}$$

$$T = 6,28 \sqrt{\frac{1m}{10 \frac{m}{s^2} \cdot (1 - 0,1)}}$$

$$T = 6,28 \sqrt{\frac{1m}{10 \frac{m}{s^2} \cdot 0,9}}$$

$$T = 6,28 \sqrt{\frac{1m}{9 \frac{m}{s^2}}}$$

$$T = 6,28 \sqrt{0,11s^2}$$

$$T = 6,28 \cdot 0,33s$$

$$T \approx 2,07s$$

$$\nu = \frac{1}{T}$$

$$\nu = \frac{1}{2,07s}$$

$$\nu \approx 0,48Hz$$

Matematičko klatno dužine 1m osciluje u vagonu. Odredi period i frekvenciju klatna ako je konstantno ubrzanje vagona $5 \frac{m}{s^2}$.

$$l = 1m$$

$$a = 5 \frac{m}{s^2}$$

$$T = ?$$

$$\nu = ?$$

$$T = 2\pi \sqrt{\frac{l}{\sqrt{g^2 + a^2}}}$$

$$T = 2 \cdot 3,14 \sqrt{\frac{1m}{\sqrt{(10 \frac{m}{s^2})^2 + (5 \frac{m}{s^2})^2}}}$$

$$T = 6,28 \sqrt{\frac{1m}{\sqrt{100 \frac{m^2}{s^4} + 25 \frac{m^2}{s^4}}}}$$

$$T = 6,28 \sqrt{\frac{1m}{\sqrt{125 \frac{m^2}{s^4}}}}$$

$$T = 6,28 \sqrt{\frac{1m}{11,18 \frac{m}{s^2}}}$$

$$T = 6,28 \sqrt{0,09s^2}$$

$$T = 6,28 \cdot 0,3s$$

$$T = 1,884s$$

$$\nu = \frac{1}{T}$$

$$\nu = \frac{1}{1,884s}$$

$$\nu \approx 0,53Hz$$